Mass and Heat Transfer in the Flow of Fluids Through Fixed and Fluidized Beds of Spherical Particles

ASHIS SEN GUPTA and GEORGE THODOS

The Technological Institute, Northwestern University, Evanston, Illinois

In 1943 Gamson et al. (7) established mass and heat transfer factors j_d and j_h for the evaporation of water into a passing air stream from the surface of spherical and cylindrical particles arranged in a packed bed. Since this initial study, considerable information concerning mass and heat transfer for the flow of gases and liquids through beds of granular solids has been reported in the literature. Because of the complexity of these processes and experimental limitations, these studies have been restricted in the range of variables considered.

Several investigators have reported mass and heat transfer factors for the evaporation of organic substances into gases from the surface of spherical particles. Hobson and Thodos (10) studied the mass transfer across a gas film by the evaporation of *n*-butanol, toluene, *n*-octane, and *n*-dodecane into streams of nitrogen, air, hydrogen, and carbon dioxide. Extensions of this study to determine mass transfer behavior across a liquid film were conducted by McCune and Wilhelm (12) for the solution of β-naphthol by water; Gaffney and Drew (5) for succinic acid and salicylic acid into acetone, benzene, and *n*-butanol; and Hobson and Thodos (9) for the transfer of isobutyl alcohol and methyl ethyl ketone into water.

Investigations have also been conducted for mass transfer into gas and liquid streams from fluidized beds of spherical particles. McCune and Wilhelm (12) determined j_d factors for the flow of water through a fluidized bed of β-naphthol particles, while Chu, Kalil, and Wetteroth (2) report values for the sublimation of naphthalene into air. Mass transfer factors have recently been reported by De Acetis and Thodos (3) for the evaporation of water from the surface of celite spheres randomly interspersed with solid spheres to produce an extended bed arrangement. In an extension of this study McConnachie (11) determined j_d factors for distended beds of celite spheres, formed by separating the spheres and holding them in place with short lengths of fine rigid wire. For these distended beds wall effects were eliminated by shaping the outer spheres of the bed to the geometry of the cylindrical reactor.

Investigations concerning heat transfer from the surface of granular particles arranged in a packed bed to a gas stream flowing through the bed have been less extensive than the corresponding mass transfer studies. Although the initial study of Gamson et al. (7) was conducted primarily to obtain information concerning mass transfer, in this investigation heat transfer factors j_h were calculated and related to the Reynolds number with the assumption that the surface temperature of the particles was the same as the wet-bulb temperature of the inlet air. Similar work in this area was later conducted by Glaser and Thodos (8) who used metallic spheres heated electrically and measured surface temperatures directly with thermocouples. Baumeister and Bennett (1) utilized steel spheres heated by high frequency induction. Recently De Acetis and Thodos (3) and McConnachie (11) have determined j_h factors for the evaporation of water into air from spheres arranged in packed, extended, and distended beds. In these investigations the sur-

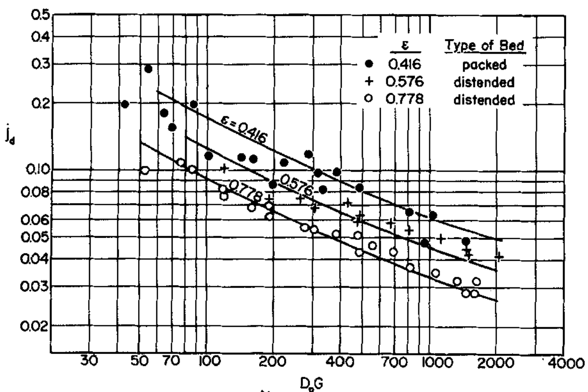

Fig. 1. Relationship between j_d and Reynolds number resulting from the mass transfer data of McConnachie for packed and distended beds of spheres.

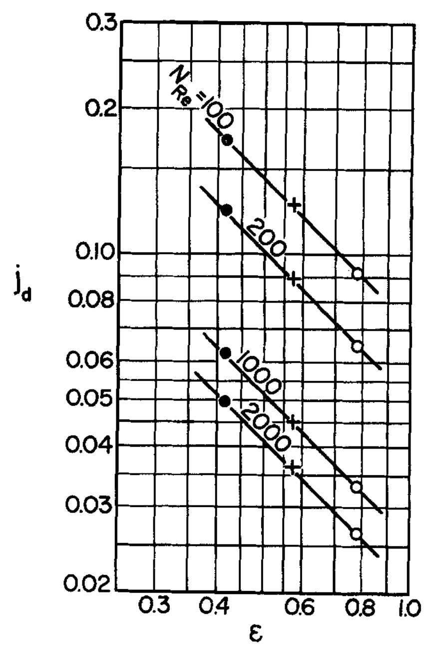

Fig. 2. Relationships between mass transfer factor j_d and void fraction ε resulting from the data of McConnachie for packed and distended beds of spheres.

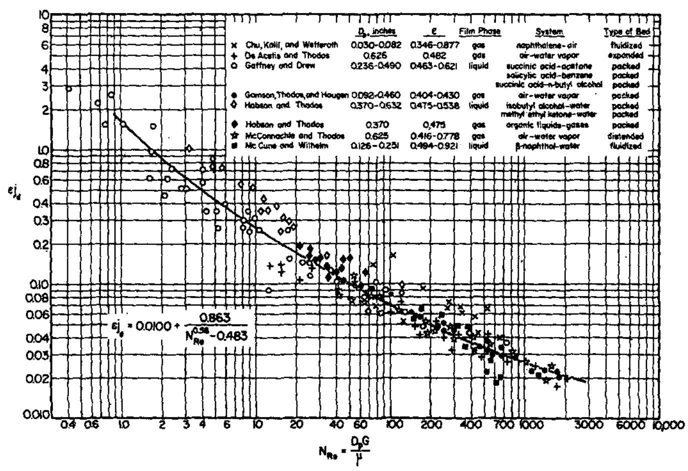

Fig. 3. Relationship between ϵj_d and Reynolds number for all types of beds of spheres.

face temperature of the spheres was measured directly with thermocouples and was found to approach the wet-bulb temperature of the inlet air only at high air velocities.

PREVIOUS CORRELATIONS OF MASS AND HEAT TRANSFER FACTORS

Several attempts have been made to correlate mass and heat transfer factors obtained from various experimental investigations. McCune and Wilhelm (*12*) attempted to relate the mass transfer group ϵj_d with the modified Reynolds number $D_pG/\epsilon\mu$, where ϵ is the void fraction of the bed, but found that the groups obtained from their data for fluidized beds did not correlate with the corresponding groups for packed beds. In 1951, Gamson (*6*), utilizing the available mass and heat transfer data for packed and fluidized beds (*7, 9, 12*), indicated that when the mass transfer factor j_d was plotted vs. the modified Reynolds group $D_pG/\mu(1-\epsilon)$, a separate curve resulted for each void fraction. In order to produce a unique relationship applicable to both packed and fluidized beds Gamson related the mass transfer modulus $j_d/(1-\epsilon)^{0.2}$ to the modified Reynolds group $D_pG/\mu(1-\epsilon)$. However there appears to be no real justification for modifying the Reynolds number by the introduction of a factor containing the void fraction. In view of this consideration an attempt has been made in the present study to correlate transfer factors with the conventional Reynolds number D_pG/μ, utilizing all of the available data including the recent information for extended and distended beds.

CORRELATION OF MASS AND HEAT TRANSFER FACTORS

In the past, difficulties have been encountered in correlating transfer factors of packed beds with those for fluidized beds. For fluidized beds the void fraction ϵ is difficult to determine accurately because of the extreme fluctuations of the bed height encountered. However the recent study of McConnachie (*11*) on mass and heat transfer into air from spheres held fixed in space in a body-centered cubic arrangement to produce a distended bed permits the establishment of the relationship between the transfer factors and the Reynolds number D_pG/μ for different void fractions of the distended fixed bed. With this distended bed arrangement void fractions comparable to those prevailing in fluidized beds are realized.

In Figure 1 the j_d factor is plotted against the Reynolds number for the packed bed and distended beds investigated by McConnachie. Separate, parallel relationships are obtained for each void fraction. For four fixed Reynolds numbers j_d was plotted against ϵ as shown in Figure 2 and produced separate parallel straight lines. The straight lines of Figure 2 were found to have slopes of -1, indicating that ϵj_d = constant for each Reynolds number, or $\epsilon j_d = f(N_{Re})$. Since the ratio of j_h/j_d has been shown (*7, 11*) to be essentially constant over a wide range of Reynolds numbers, a similar dependence between ϵj_h and the Reynolds number is indicated.

TREATMENT OF MASS AND HEAT TRANSFER DATA

Experimental mass and heat transfer data for all types of beds reported in the literature were utilized to determine the exact dependences between the products ϵj_d and ϵj_h and the Reynolds number D_pG/μ. For mass transfer the data of the investigations for gas and liquid films in packed, fluidized, expanded, and distended beds were used. The data of Gaffney and Drew (*5*) were particularly important in establishing j_d values in the low Reynolds region. In Figure 3 representative ϵj_d values obtained from the eight basic sources were plotted against the corresponding Reynolds number. Although some scatter is encountered, particularly at low Reynolds number where back-mixing effects are prevalent, a single relationship appears to exist between ϵj_d and D_pG/μ which is independent of the type of bed. The relationship representing the data best can be expressed by the equation

$$\epsilon j_d = 0.010 + \frac{0.863}{N_{Re}^{0.58} - 0.483} \quad \text{for } N_{Re} > 1 \qquad (1)$$

Equation (1) is restricted to the region $N_{Re} > 1$ because only meager mass transfer data are available for lower Reynolds numbers. Although no experimental j_d values have been reported for $N_{Re} > 2{,}140$, the trend of the data in this vicinity indicates that Equation (1) should accurately represent the dependence between ϵj_d and N_{Re} in this region.

Similarly the products of heat transfer factors obtained from the literature

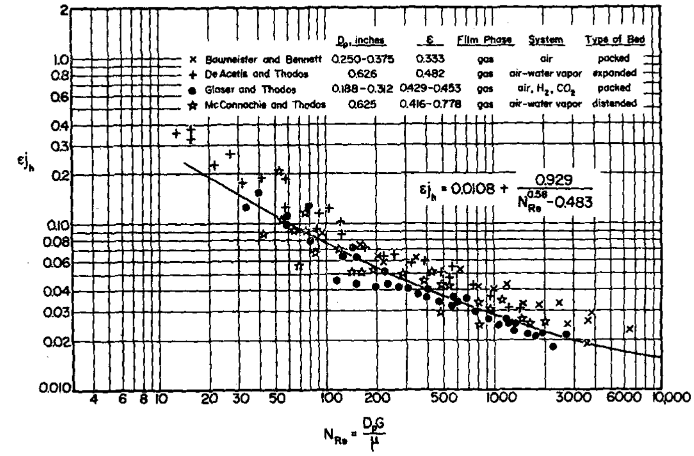

Fig. 4. Relationship between ϵj_h and Reynolds number for beds of spheres.

and the void fraction of the bed were plotted against the corresponding Reynolds number, as shown in Figure 4. Only the data of studies in which surface temperatures were measured directly were utilized and include j_h values for packed, expanded, and distended beds. No experimental heat transfer factors are presently available for fluidized beds. Again considerable scatter is noted at low Reynolds numbers, due to backmixing effects. The deviations of the j_h values reported by De Acetis and Thodos (*3*) can be attributed to the significant conduction and radiation effects which were not accounted for in that study. The best relationship, which is limited to the region $N_{Re} > 20$ for which substantial experimental data are available, may be expressed analytically as follows:

$$\epsilon j_h = 0.0108 + \frac{0.929}{N_{Re}^{0.58} - 0.483} \quad \text{for } N_{Re} > 20 \qquad (2)$$

Dividing Equation (2) by Equation (1) for the region $N_{Re} > 20$ where both equations apply, one obtains $\frac{\epsilon j_h}{\epsilon j_d} = \frac{j_h}{j_d} = 1.076$, which is consistent with the previously reported values (*7, 11*).

Evnochides and Thodos (*4*) have established mass and heat transfer factors for the flow of air past a single sphere. For this case the void fraction is equal to unity. Transfer factors calculated from Equations (*1*) and (*2*) with $\epsilon = 1$ are consistent with the corresponding experimental values obtained by Evnochides and Thodos for $N_{Re} \approx 2{,}000$, which is the lowest Reynolds number considered in their study and the highest value utilized in the present investigation. Therefore Equations (*1*) and (*2*) do not have the disadvantage of the previous correlations which utilized the factor $(1 - \epsilon)$ and thus could not be extended to include the limiting case of flow past a single sphere.

Values of mass and heat transfer factors calculated with Equations (*1*) and (*2*) were compared with the corresponding values obtained from experimental data. Equation (*1*) was found to reproduce 380 j_d values reported by eight investigators with an average deviation of 15.6% while Equation (2) reproduced 168 j_h values reported by four investigators with an average deviation of 17.1%. The average deviation is defined as the ratio of the difference between the value reported in the literature and calculated to the value reported in the literature. In these comparisons j factors were obtained from experimental data from several sources which appear to be inconsistent and in which experimental errors are indicated. Thus these average deviations are quite probably higher than those which might have resulted if these questionable values had been eliminated in the comparisons.

NOTATION

c_p = heat capacity, B.t.u./lb. °F.
c_{if} = concentration of nontransferable component, lb. moles/ cu. ft.
D_p = particle diameter, ft.
D_l = diffusivity of transferable component in liquid phase, sq.ft./ hr.
D_v = diffusivity of transferable component in gas phase, sq.ft./hr.
G = superficial mass velocity of flowing gas, lb./hr. sq.ft.
h = heat transfer coefficient, B.t.u./ sq.ft. °F.
j_d = mass transfer factor, $\frac{k_g p_{gf}}{G/M} \left(\frac{\mu}{\rho D_v}\right)_f^{2/3}$; $\frac{k_l c_{if}}{L/M} \left(\frac{\mu}{\rho D_l}\right)_f^{2/3}$
j_h = heat transfer factor $\frac{h}{c_p G} \left(\frac{c_p \mu}{k}\right)_f^{2/3}$
k = thermal conductivity, B.t.u./ hr. ft. °F.
k_g = mass transfer coefficient for gas film, lb. moles/hr. sq.ft. atm.
k_l = mass transfer coefficient for liquid film, lb. moles/hr. sq.ft. $\frac{\text{lb. mole}}{\text{cu.ft.}}$
L = superficial mass velocity of flowing liquid, lb/hr. sq.ft.
M = molecular weight of flowing stream, lb./lb. mole
N_{Re} = Reynolds number D_pG/μ
p_{gf} = partial pressure of nontransferable component in gas film, atm
ϵ = void fraction of bed
μ = viscosity, lb./hr.ft.
ρ = density, lb./cu.ft.

LITERATURE CITED

1. Baumeister, B. E., and C. O. Bennett, *A.I.Ch.E. Journal*, **4**, 69 (1958).
2. Chu, J. C., James Kalil, and W. A. Wetteroth, *Chem. Eng. Progr.*, **49**, 141 (1953).
3. De Acetis, James, and George Thodos, *Ind. Eng. Chem.*, **52**, 1003 (1960).
4. Evnochides, Spyros, and George Thodos, *A.I.Ch.E. Journal*, **7**, 78 (1961).
5. Gaffney, B. J., and T. B. Drew, *Ind. Eng. Chem.*, **42**, 1120 (1950).
6. Gamson, B. W., *Chem. Eng. Progr.*, **47**, 19 (1951).
7. ———, George Thodos, and O. A. Hougen, *Trans. Am. Inst. Chem. Engrs.*, **39**, 1 (1943).
8. Glaser, M. B., and George Thodos, *A.I.Ch.E. Journal*, **4**, 63 (1958).
9. Hobson, Merk, and George Thodos, *Chem. Eng. Progr.*, **45**, 517 (1949).
10. *Ibid.*, **47**, 370 (1951).
11. McConnachie, J. T. L., M.S. thesis, Northwestern Univ., Evanston, Illinois (1961).
12. McCune, L. K., and R. H. Wilhelm, *Ind. Eng. Chem.*, **41**, 1124 (1949).

Manuscript received June 19, 1961; revision received September 29, 1961; paper accepted October 5, 1961.

Thermal Conductivity of Nonassociated Liquids

A. A. BONDI

Shell Development Company, Emeryville, California

The thermal conductivity of liquids composed of nonionized molecules covers a comparatively small range of numerical values, the largest and the smallest known values differing by less than a factor of 10. Yet heat transfer coefficients are sufficiently sensitive to the magnitude of the thermal conductivity (which is very difficult to meas-